

Exercise 16

Differentiate.

$$f(t) = te^t \cot t$$

SolutionUse the product and quotient rules to differentiate $f(t)$.

$$\begin{aligned} f'(t) &= \frac{d}{dt}[f(t)] \\ &= \frac{d}{dt} \left(\frac{te^t}{\tan t} \right) \\ &= \frac{\left[\frac{d}{dt}(te^t) \right] \tan t - \left[\frac{d}{dt}(\tan t) \right] te^t}{(\tan t)^2} \\ &= \frac{\left\{ \left[\frac{d}{dt}(t) \right] e^t + t \left[\frac{d}{dt}(e^t) \right] \right\} \tan t - (\sec^2 t)te^t}{\tan^2 t} \\ &= \frac{[(1)e^t + t(e^t)] \tan t - (\sec^2 t)te^t}{\tan^2 t} \\ &= \frac{(1+t)e^t \tan t - te^t \sec^2 t}{\tan^2 t} \end{aligned}$$